Call-by-Value Typing Revisited, for Free?

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Call-by-Name and Call-by-Value

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

- ✓ Well studied
- ✗ Not used

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

- ✓ Well studied
- ✗ Not used

Call-by-Value

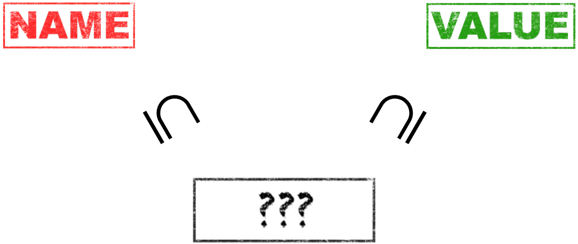
- ✗ Not understood
- ✓ Very much used

Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Call-by-Value

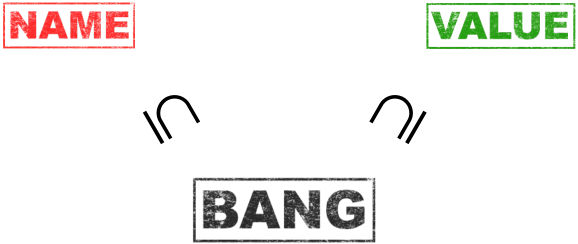


Call-by-Name and Call-by-Value

Different Models of Computation:

Call-by-Name

Call-by-Value



Preservations

Preservations

NAME t verifies P

VALUE t verifies P

Preservations

NAME $\quad t$ verifies $P \Rightarrow t^{N}$ verifies P

VALUE $\quad t$ verifies $P \Rightarrow t^{V}$ verifies P

BANG

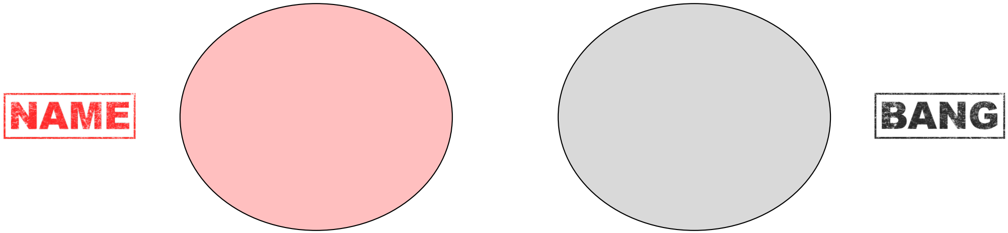
Preservations

NAME t verifies P $\Leftrightarrow$ t^{N} verifies P

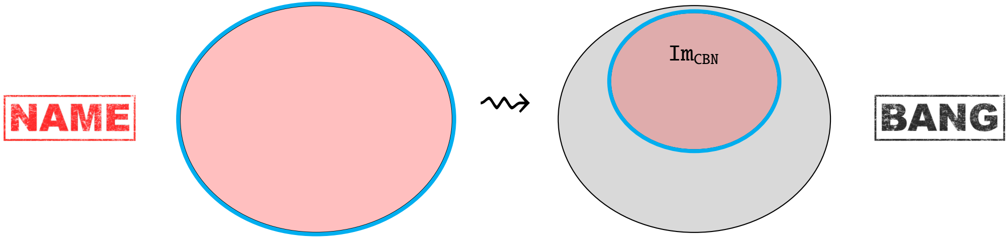
VALUE t verifies P $\Leftrightarrow$ t^{V} verifies P

BANG

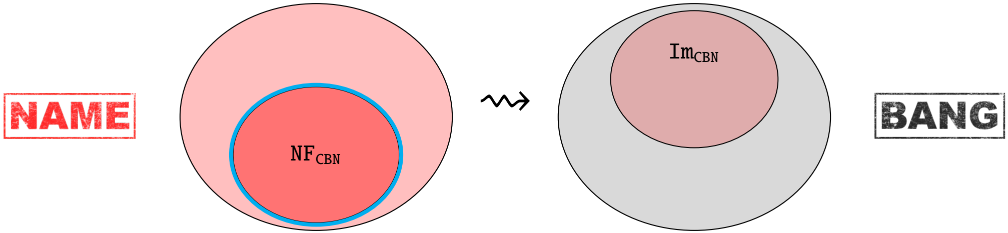
Call-by-Name Preservations



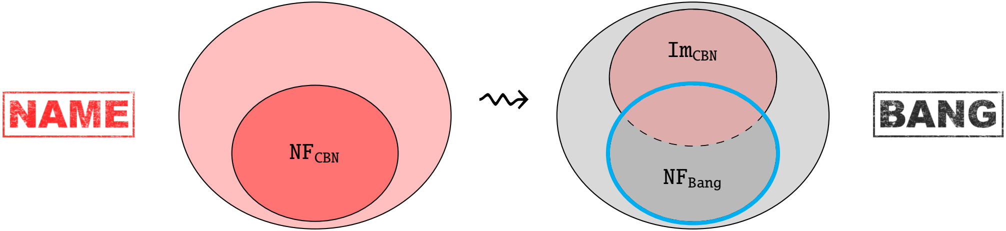
Call-by-Name Preservations

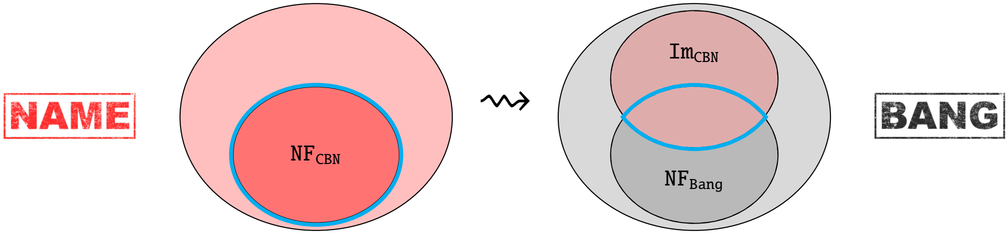


Call-by-Name Preservations



Call-by-Name Preservations

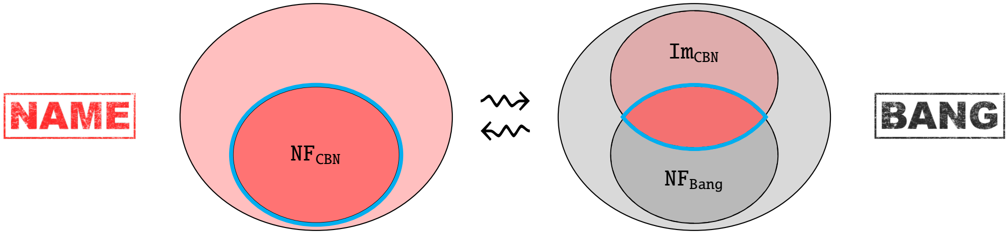




Static Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

NAME t normal form $\Rightarrow$ t^N normal form BANG

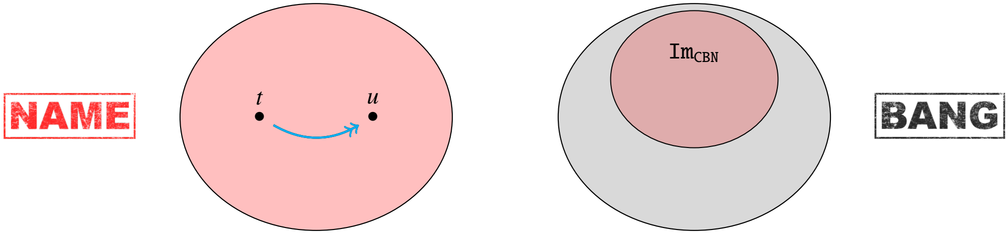
Call-by-Name Preservations



Static Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Call-by-Name Preservations



Static Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

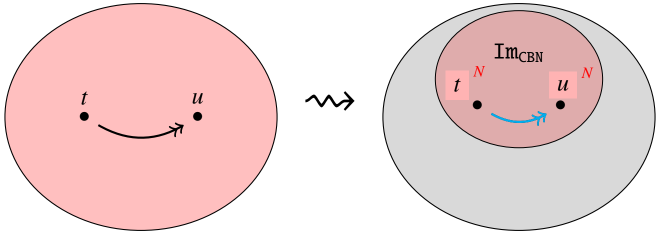
NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

Dynamic Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23, ArrialGuerrKesner'24]

NAME $t \twoheadrightarrow u$

Call-by-Name Preservations

NAME



BANG

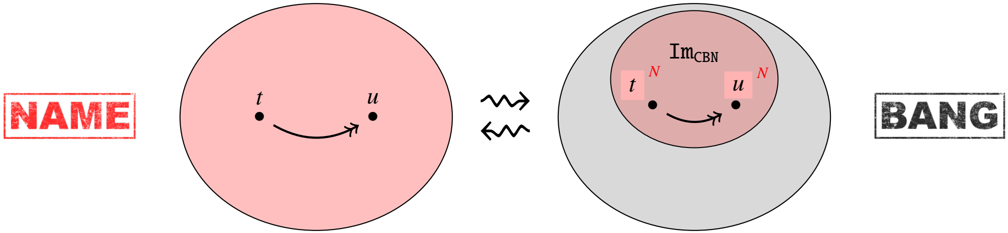
Static Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^N normal form BANG

Dynamic Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23, ArrialGuerrKesner'24]

NAME $t \twoheadrightarrow u$ $\Rightarrow$ $t^N \twoheadrightarrow u^N$ BANG

Call-by-Name Preservations



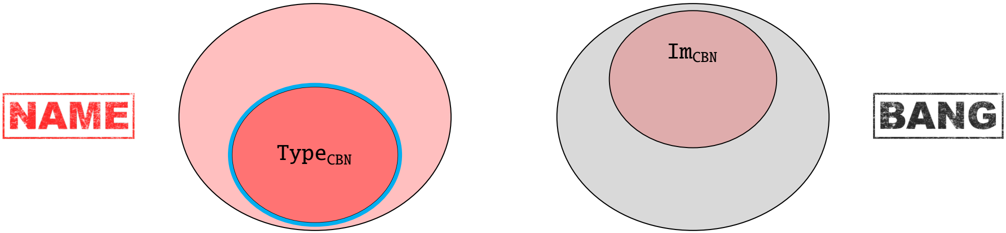
Static Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

NAME $\quad t$ normal form $\quad\Leftrightarrow\quad t^{N}$ normal form $\quad$ BANG

Dynamic Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23, ArrialGuerrKesner'24]

NAME $\quad t \twoheadrightarrow u \quad\Leftrightarrow\quad t^{N} \twoheadrightarrow u^{N}$ $\quad$ BANG

Call-by-Name Preservations



Static Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

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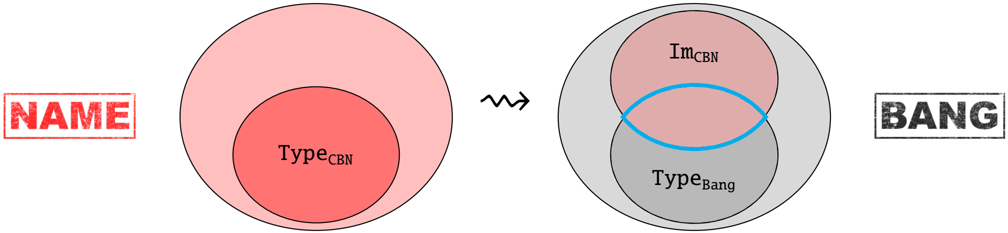
Dynamic Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23, ArrialGuerrKesner'24]

NAME $\quad t \twoheadrightarrow u \quad \Leftrightarrow \quad t^{N} \twoheadrightarrow u^{N}$ $\quad$ BANG

Typed Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

NAME $\quad \Gamma \vdash t : \sigma$

Call-by-Name Preservations



Static Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form BANG

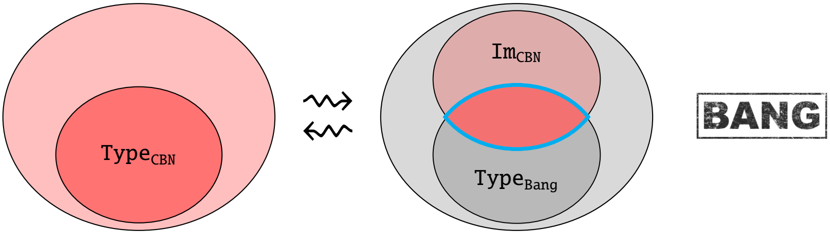
Dynamic Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23, ArrialGuerrKesner'24]

NAME $t \twoheadrightarrow u$ $\Leftrightarrow$ $t^{N} \twoheadrightarrow u^{N}$ BANG

Typed Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

NAME $\Gamma \vdash t : \sigma$ $\Rightarrow$ $\Gamma \vdash t^{N} : \sigma$ BANG

Call-by-Name Preservations



Static Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

NAME t normal form $\Leftrightarrow$ t^{N} normal form   BANG

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NAME $t \twoheadrightarrow u$ $\Leftrightarrow$ $t^{N} \twoheadrightarrow u^{N}$ BANG

Typed Properties: [GuerrManz'18, BucciaKesnerRíosViso'20,'23]

NAME $\Gamma \vdash t : \sigma$ $\Leftrightarrow$ $\Gamma \vdash t^{N} : \sigma$ BANG

A Unifying Framework: Bang-Calculus

[EhrhGuer'16][GuerManz'18]

Bang-Calculus

BANG

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

Bang-Calculus

BANG

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$

$\mid\ !t$ (value)

Bang-Calculus

BANG

(Terms)	t, u	$::=$	$x \mid \lambda x.t \mid t\,u$	
			$\mid\ !t$	(value)
			$\mid \mathtt{der}(t)$	(computation)

A Unifying Framework: Bang-Calculus [EhrhGuer'16][GuerManz'18]

Bang-Calculus

BANG

(Terms)	t, u	$::=$	$x \mid \lambda x.t \mid t\,u$	
			$\mid\ !t$	(value)
			$\mid \texttt{der}(t)$	(computation)

$$(\lambda x.t)\ !u$$

A Unifying Framework: Bang-Calculus

[EhrhGuer'16][GuerManz'18]

Bang-Calculus

(Terms)	t, u	$::=$	$x \mid \lambda x.t \mid t\,u$	
			$\mid\ !t$	(value)
			$\mid\ \mathtt{der}(t)$	(computation)

$$(\lambda x.t)\ !u \quad \mapsto_\beta \quad t\{x := u\}$$

A Unifying Framework: Bang-Calculus [EhrhGuer'16][GuerManz'18]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad \text{(value)} \\
 & & & \mid \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t)$$

A Unifying Framework: Bang-Calculus [EhrhGuer'16][GuerManz'18]

Bang-Calculus

BANG

(Terms)	t, u ::=	$x \mid \lambda x.t \mid t\,u$	
		$\mid\ !t$	(value)
		$\mid \mathtt{der}(t)$	(computation)

$$(\lambda x.t)\,!u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \mapsto_{!} \quad t$$

A Unifying Framework: Bang-Calculus

[EhrhGuer'16][GuerManz'18]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad \text{(value)} \\
 & & & \mid\ \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\ !u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \mapsto_{!} \quad t$$

$(\lambda x.x!x)\ {\color{red}(\mathtt{I}\ !\mathtt{I})}$

A Unifying Framework: Bang-Calculus [EhrhGuer'16][GuerManz'18]

Bang-Calculus

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
 & & & \mid\ \mathtt{der}(t) \qquad\qquad\quad\; \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \mapsto_! \quad t$$

$$(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) \quad \rightarrow \quad (\lambda x.x!x)\,!\mathtt{I}$$

A Unifying Framework: Bang-Calculus [EhrhGuer'16][GuerManz'18]

Bang-Calculus

BANG

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\; !t \qquad \text{(value)} \\
 & & & \mid\; \mathtt{der}(t) \qquad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\,!u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \mapsto_! \quad t$$

$$(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) \quad \to \quad (\lambda x.x!x)\,!\mathtt{I} \quad \to \quad \mathtt{I}!\mathtt{I} \quad \to \quad !\mathtt{I}$$

A Unifying Framework: Bang-Calculus [EhrhGuer'16][GuerManz'18]

Bang-Calculus

BANG

$$\textbf{(Terms)} \quad t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u$$

$$\mid\ !t \qquad \text{(value)}$$

$$\mid \texttt{der}(t) \qquad \text{(computation)}$$

$$(\lambda x.t)\,!u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\texttt{der}(!t) \quad \mapsto_{!} \quad t$$

$$(\lambda x.x!x)\,(\texttt{I}\,!\texttt{I}) \quad \rightarrow \quad (\lambda x.x!x)\,!\texttt{I} \quad \rightarrow \quad \texttt{I}!\texttt{I} \quad \rightarrow \quad !\texttt{I}$$

$$(\lambda x.y)\,\Omega$$

A Unifying Framework: Bang-Calculus [EhrhGuer'16][GuerManz'18]

Bang-Calculus

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
 & & & \mid \mathtt{der}(t) \qquad\qquad\quad \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\ !u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \mapsto_{!} \quad t$$

$$
\begin{array}{rclclcl}
(\lambda x.x!x)\,(\mathtt{I}\ !\mathtt{I}) & \to & (\lambda x.x!x)\ !\mathtt{I} & \to & \mathtt{I}!\mathtt{I} & \to & !\mathtt{I} \\
(\lambda x.y)\ \color{red}{\Omega} & \to & (\lambda x.y)\ \Omega & & & &
\end{array}
$$

A Unifying Framework: Bang-Calculus [EhrhGuer'16][GuerManz'18]

Bang-Calculus

$$
\begin{array}{llll}
\textbf{(Terms)} & t, u & ::= & x \mid \lambda x.t \mid t\,u \\
 & & & \mid\ !t \qquad\qquad\qquad\qquad \text{(value)} \\
 & & & \mid \mathtt{der}(t) \qquad\qquad\quad\ \text{(computation)}
\end{array}
$$

$$(\lambda x.t)\ !u \quad \mapsto_{\beta} \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \mapsto_{!} \quad t$$

$$
\begin{array}{ccccccc}
(\lambda x.x!x)\,(\mathtt{I}\ !\mathtt{I}) & \to & (\lambda x.x!x)\ !\mathtt{I} & \to & \mathtt{I}!\mathtt{I} & \to & !\mathtt{I} \\
(\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & \cdots
\end{array}
$$

A Unifying Framework: Bang-Calculus [EhrhGuer'16][GuerManz'18]

Bang-Calculus

BANG

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x \backslash u]$

$\mid\ !t$ (value)

$\mid \mathtt{der}(t)$ (computation)

$$(\lambda x.t)\ !u \quad \mapsto_\beta \quad t\{x := u\}$$

$$\mathtt{der}(!t) \quad \mapsto_! \quad t$$

$$(\lambda x.x!x)\,(\mathtt{I}\ !\mathtt{I}) \quad \to \quad (\lambda x.x!x)\ !\mathtt{I} \quad \to \quad \mathtt{I}!\mathtt{I} \quad \to \quad !\mathtt{I}$$

$$(\lambda x.y)\,\Omega \quad \to \quad (\lambda x.y)\,\Omega \quad \to \quad (\lambda x.y)\,\Omega \quad \to \quad \cdots$$

A Unifying Framework: Bang-Calculus [BucciaKesnerRíosViso’20’23]

Bang-Calculus

BANG

(Terms) $t, u \quad ::= \quad x \mid \lambda x.t \mid t\,u \mid t[x\backslash u]$

$\quad\quad\quad\quad \mid\ !t$ (value)

$\quad\quad\quad\quad \mid\ \mathtt{der}(t)$ (computation)

$$
\begin{array}{rcl}
L\langle \lambda x.t\rangle\, !u & \mapsto_{\beta} & L\langle t[x\backslash u]\rangle \\
t[x\backslash L\langle !u\rangle] & \mapsto_{s!} & L\langle t\{x := u\}\rangle \\
\mathtt{der}(L\langle !t\rangle) & \mapsto_{!} & L\langle t\rangle
\end{array}
$$

$$
\begin{array}{rcccccccc}
(\lambda x.x!x)\,(\mathtt{I}\,!\mathtt{I}) & \to & (\lambda x.x!x)\,!\mathtt{I} & \to & \mathtt{I}!\mathtt{I} & \to & !\mathtt{I} & & \\
(\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & (\lambda x.y)\,\Omega & \to & \cdots & &
\end{array}
$$

Call-by-Value Embeddings [GuerrieriManzonetto'18]

t^{V} : VALUE $\longrightarrow$ BANG

$$t^{V} : \textbf{VALUE} \longrightarrow \textbf{BANG}$$

$$x^{V} := x$$

$$(\lambda x.t)^{V} := \lambda x. t^{V}$$

$$(t\,u)^{V} := t^{V} \; u^{V}$$

Call-by-Value Embeddings

[GuerrieriManzonetto'18]

t^V : VALUE $\longrightarrow$ BANG

$$x^V := !x$$

$$(\lambda x.t)^V := !\lambda x.\, t^V$$

$$(t\,u)^V := t^V \quad u^V$$

Call-by-Value Embeddings

[GuerrieriManzonetto'18]

$$t^V : \quad \textbf{VALUE} \longrightarrow \textbf{BANG}$$

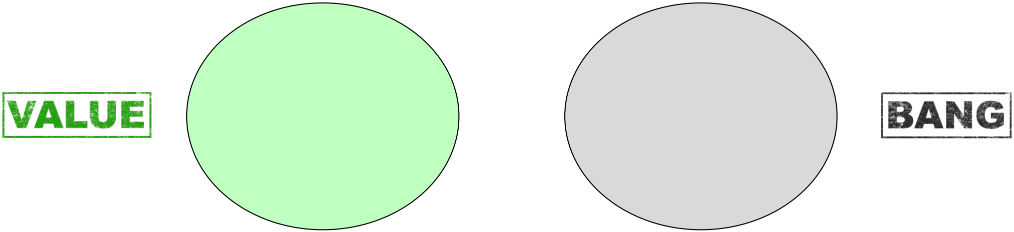
$$
\begin{aligned}
x^V &:= \; !x \\
(\lambda x.t)^V &:= \; !\lambda x.\, t^V \\
(t\,u)^V &:= \; \mathtt{der}(t^V)\; u^V
\end{aligned}
$$

Call-by-Value Embeddings [GuerrieriManzonetto'18]

t^{V} : **VALUE** $\longrightarrow$ **BANG**

$$
\begin{aligned}
x^{V} &:= \,!x \\
(\lambda x.t)^{V} &:= \,!\lambda x.\, t^{V} \\
(t\,u)^{V} &:= \mathtt{der}(t^{V})\; u^{V} \\
(t[x\backslash u])^{V} &:= t^{V}[x\backslash u^{V}]
\end{aligned}
$$

Call-by-Value Preservations



Static Properties:

VALUE — t normal form — t^V normal form — BANG

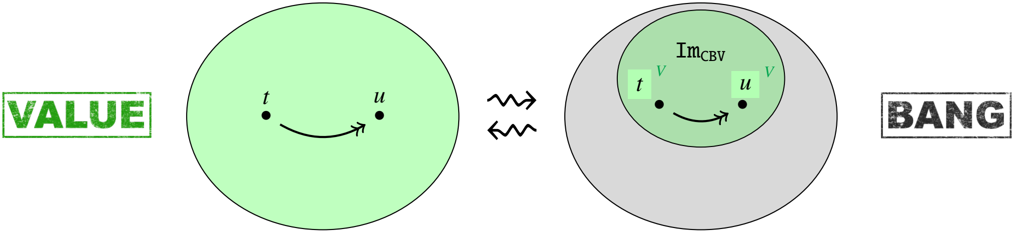
Dynamic Properties: [GuerrieriManzonetto'18]

VALUE — $t \twoheadrightarrow u$ — $t^V \twoheadrightarrow u^V$ — BANG

Typed Properties: [GuerrieriManzonetto'18]

VALUE — $\Gamma \vdash t : \sigma$ — $\Gamma \vdash t^V : \sigma$ — BANG

Call-by-Value Preservations



Static Properties:

VALUE t normal form t^{V} normal form   BANG

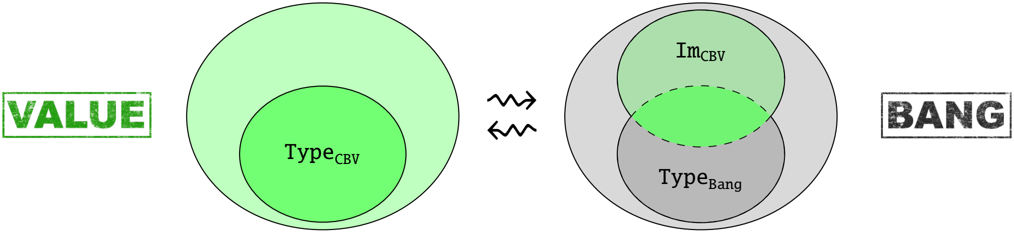
Dynamic Properties: [GuerrieriManzonetto'18]

VALUE $t \twoheadrightarrow u$ $\Leftrightarrow$ $t^{V} \twoheadrightarrow u^{V}$ BANG

Typed Properties: [GuerrieriManzonetto'18]

VALUE $\Gamma \vdash t : \sigma$ $\Gamma \vdash t^{V} : \sigma$ BANG

Call-by-Value Preservations



Static Properties:

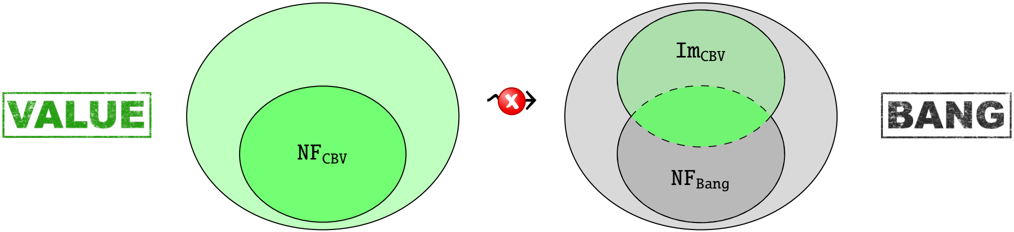
VALUE t normal form $\quad t^{V}$ normal form BANG

Dynamic Properties: [GuerrieriManzonetto'18]

VALUE $t \twoheadrightarrow u \quad \Leftrightarrow \quad t^{V} \twoheadrightarrow u^{V}$ BANG

Typed Properties: [GuerrieriManzonetto'18]

VALUE $\Gamma \vdash t : \sigma \quad \Leftrightarrow \quad \Gamma \vdash t^{V} : \sigma$ BANG



Static Properties:

VALUE t normal form $\not\Rightarrow$ t^V normal form BANG

Dynamic Properties: [GuerrieriManzonetto'18]

VALUE $t \twoheadrightarrow u$ $\Leftrightarrow$ $t^V \twoheadrightarrow u^V$ BANG

Typed Properties: [GuerrieriManzonetto'18]

VALUE $\Gamma \vdash t : \sigma$ $\Leftrightarrow$ $\Gamma \vdash t^V : \sigma$ BANG

Counterexamples

$x\,y$

VALUE

BANG


x y
VALUE
BANG

Counterexamples

Counterexamples

$$x\,y \quad \rightsquigarrow \quad \texttt{der}(x^{V})\, y^{V}$$

VALUE

BANG

Counterexamples

$$x\,y \quad \rightsquigarrow \quad \texttt{der}(!x)\ \boxed{y}^{V}$$

VALUE

BANG

Counterexamples

$$x\,y \quad \rightsquigarrow \quad \mathtt{der}(!x)\,(!y)$$

Counterexamples

Call-by-Value Embeddings

[GuerrieriManzonetto'18]

t^{V} : **VALUE** $\longrightarrow$ **BANG**

$$
\begin{aligned}
x^{V} &:= \,!x \\
\lambda x.t^{V} &:= \,!\lambda x.t^{V} \\
t\,u^{V} &:= \mathtt{der}(t^{V})\,u^{V} \\
t[x\backslash u]^{V} &:= t^{V}[x\backslash u^{V}]
\end{aligned}
$$

t^V : **VALUE** $\longrightarrow$ **BANG**

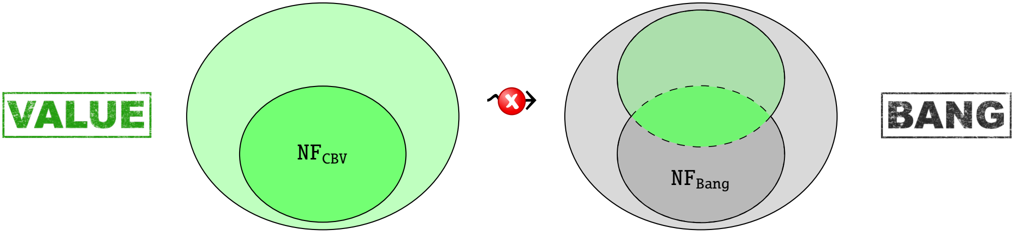
$$x^V := !x$$

$$(\lambda x.t)^V := !\lambda x.t^V$$

$$(t\,u)^V := \mathtt{der}(t^V)\,u^V \quad +\mathtt{superdevelopment}$$

$$(t[x\backslash u])^V := t^V[x\backslash u^V]$$

Call-by-Value Preservations



Static Properties:

VALUE t normal form $\not\Rightarrow$ t^{V} normal form BANG

Dynamic Properties: [GuerrieriManzonetto'18]

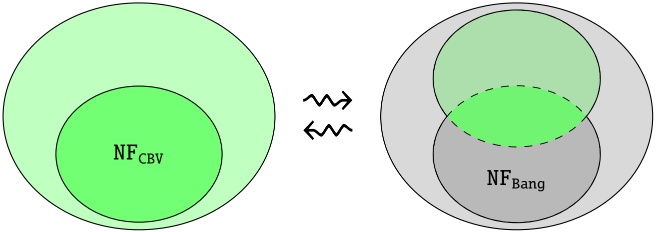
VALUE $t \twoheadrightarrow u$ $\Leftrightarrow$ $t^{V} \twoheadrightarrow u^{V}$ BANG

Typed Properties: [GuerrieriManzonetto'18]

VALUE $\Gamma \vdash t : \sigma$ $\Leftrightarrow$ $\Gamma \vdash t^{V} : \sigma$ BANG

Call-by-Value Preservations

VALUE



BANG

Static Properties:

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

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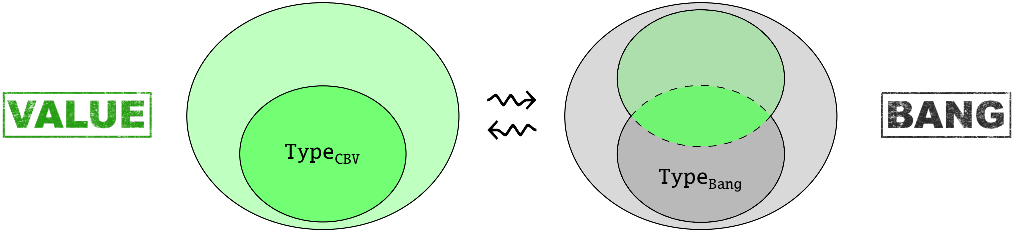
VALUE $\Gamma \vdash t : \sigma$ $\Leftrightarrow$ $\Gamma \vdash t^{V} : \sigma$ BANG

Call-by-Value Preservations



Static Properties:

VALUE t normal form $\Leftrightarrow$ t^{V} normal form BANG

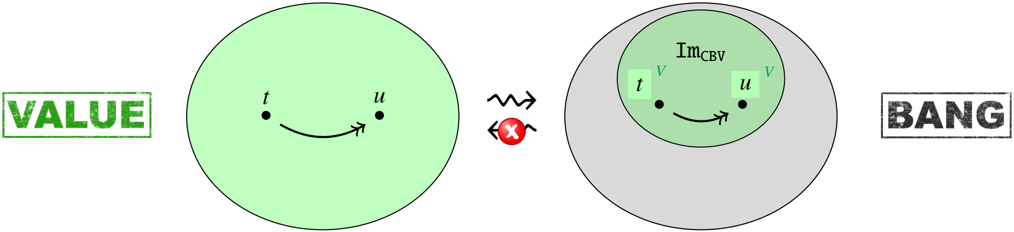
Dynamic Properties: [GuerrieriManzonetto'18]

VALUE $t \twoheadrightarrow u$ $\Leftrightarrow$ $t^{V} \twoheadrightarrow u^{V}$ BANG

Typed Properties: [GuerrieriManzonetto'18]

VALUE $\Gamma \vdash t : \sigma$ $\Leftrightarrow$ $\Gamma \vdash t^{V} : \sigma$ BANG

Call-by-Value Preservations



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Counterexamples

$$(\lambda x.\Omega)\, y$$

VALUE **BANG**

Counterexamples

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad \boxed{(\lambda x.\Omega)\, y}^{V}$$

VALUE

BANG

Counterexamples

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad \mathtt{der}(\boxed{\lambda x.\Omega}^{V})\, \boxed{y}^{V}$$

VALUE

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Counterexamples

$$(\lambda x.\Omega)\,y \quad \rightsquigarrow \quad \mathtt{der}(!\lambda x.\,\boxed{\Omega}^{V})\,\boxed{y}^{V}$$

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VALUE **BANG**

Counterexamples

$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x. \Omega^{V})\, !y$

VALUE

$\downarrow$

BANG

$(\lambda x.(z!z)[z \backslash ! \Delta^{V}])\, (!y)$

Counterexamples

$$(\lambda x.\Omega)\, y \quad \rightsquigarrow \quad (\lambda x. \Omega^{V})\, !y$$

VALUE $\qquad\qquad \downarrow \qquad\qquad$ **BANG**

$$(\lambda x.(zz)[z\backslash\Delta])\, y \quad \rightsquigarrow \quad (\lambda x.(z!z)[z\backslash !\Delta^{V}])\,(!y)$$

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VALUE $\downarrow$ (✗) $\qquad\qquad\qquad\qquad$ $\downarrow$ BANG

$$(\lambda x.(zz)[z\backslash\Delta])\, y \quad \rightsquigarrow \quad (\lambda x.(z!z)[z\backslash !\Delta^{V}])\,(!y)$$

t^V : **VALUE** $\longrightarrow$ **BANG**

$$
\begin{array}{rcl}
x^V & := & !x \\
(\lambda x.t)^V & := & !\lambda x.\, t^V \\
(t\,u)^V & := & \mathtt{der}(t^V)\; u^V \quad + \mathtt{superdevelopment} \\
(t[x\backslash u])^V & := & t^V[x\backslash u^V]
\end{array}
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Call-by-Value Embeddings

[ArrialGuerrieriKesner'24]

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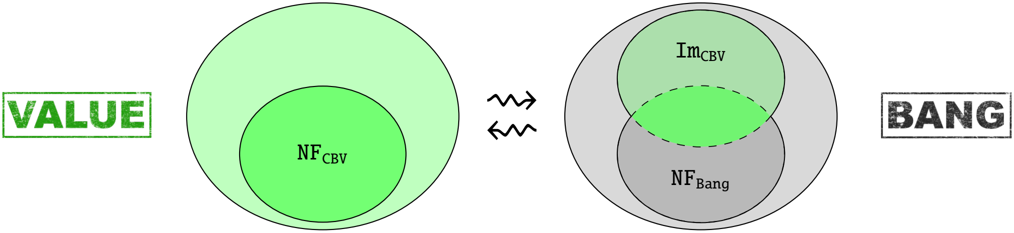

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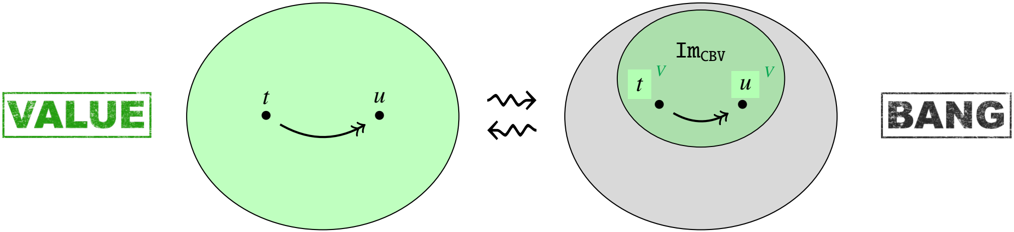

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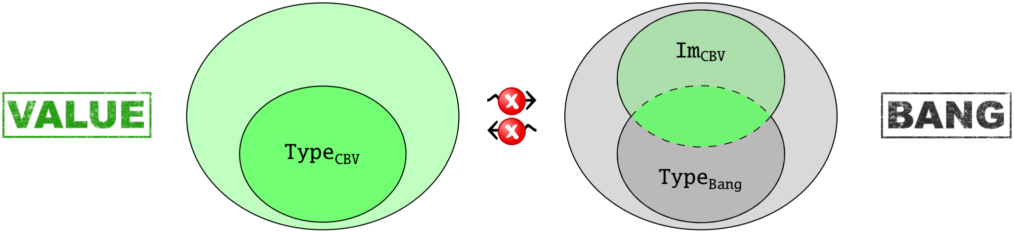

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Call-by-Value Quantitative Typing System

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$$\frac{}{x : \mathcal{M} \vdash x : \mathcal{M}} \ (\texttt{var})$$

$$\frac{\Gamma_1 \vdash t : [\mathcal{M} \Rightarrow \sigma] \qquad \Gamma_2 \vdash u : \mathcal{M}}{\Gamma_1 + \Gamma_2 \vdash t\,u : \sigma} \ (\texttt{app})$$

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$$\frac{}{\emptyset \vdash \lambda x.t : [\,]} \; (\texttt{abs}) \qquad \frac{\Gamma_1; x : \mathcal{M} \vdash t : \sigma \qquad \Gamma_2 \vdash u : \mathcal{M}}{\Gamma_1 + \Gamma_2 \vdash t[x \backslash u] : \sigma} \; (\texttt{es})$$

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Theorem ([BucciarelliKesnerRíosViso'20'23])

Let $t \in \Lambda$, then t is $\rightarrow$-normalizing iff it is $\mathcal{V}$-typable.

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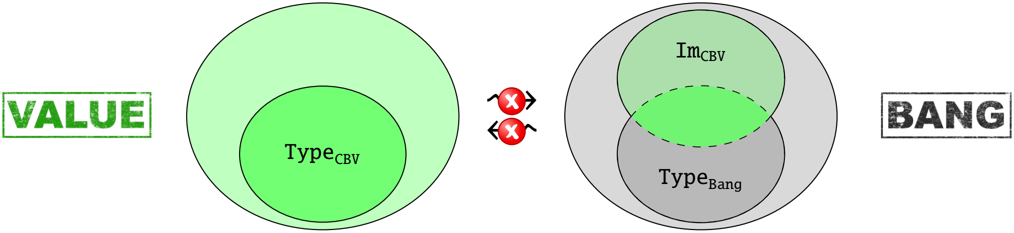

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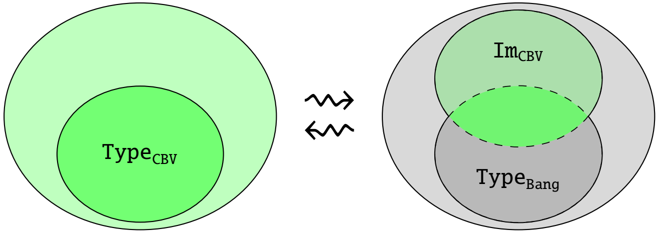

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Let t *be a* *normal form*, *then there exists* $\Pi \triangleright_{\mathcal{V}'} \Gamma \vdash t : \sigma$ *for some* Γ, σ.

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Lemma (Weighted Subject Reduction)

Let $t \to u$ *with* $\Pi \triangleright_{\mathcal{V}'} \Gamma \vdash t : \sigma$. *Then there exists* $\Pi' \triangleright_{\mathcal{V}'} \Gamma \vdash u : \sigma$ *such that* $\#(\Pi) > \#(\Pi')$.

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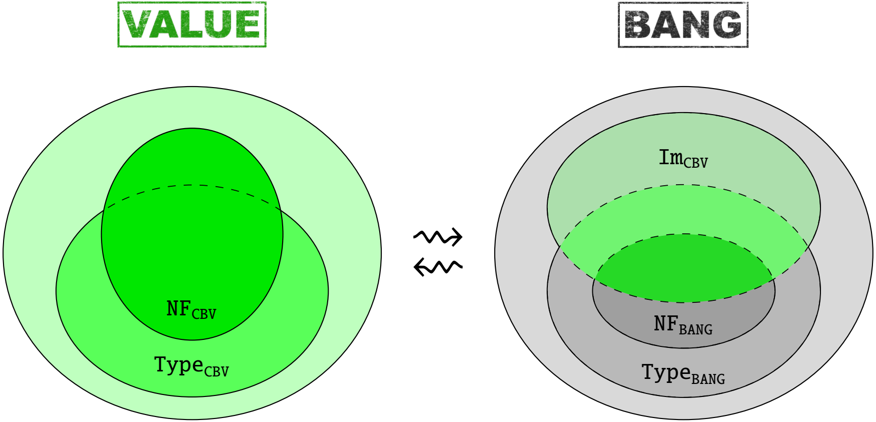

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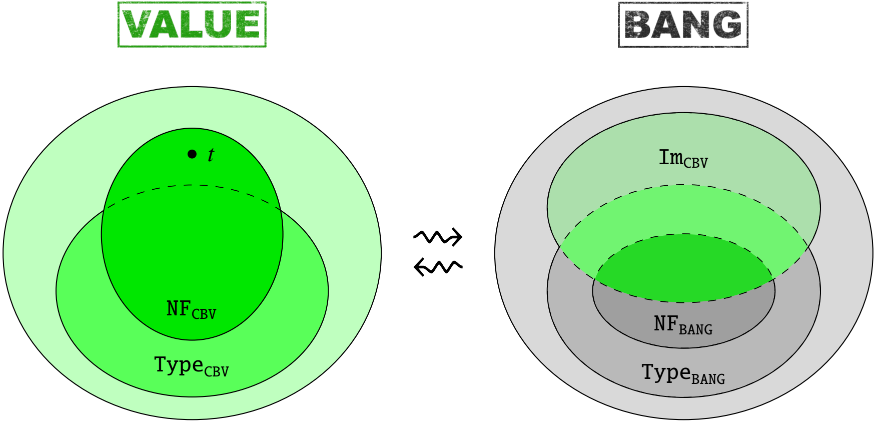

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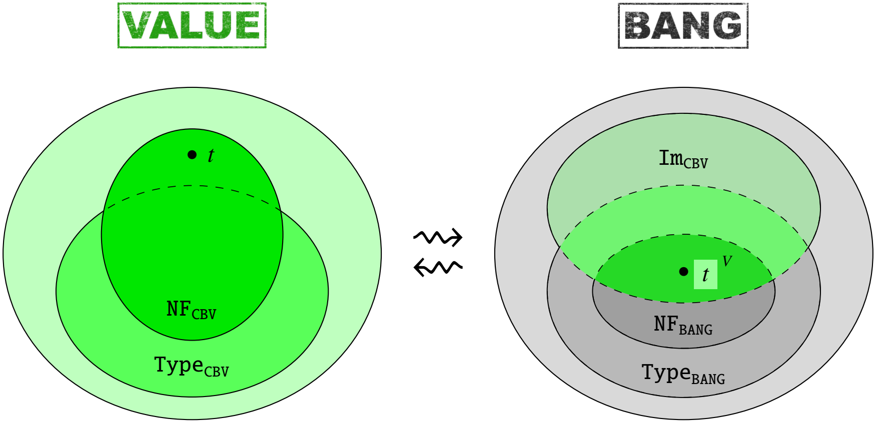

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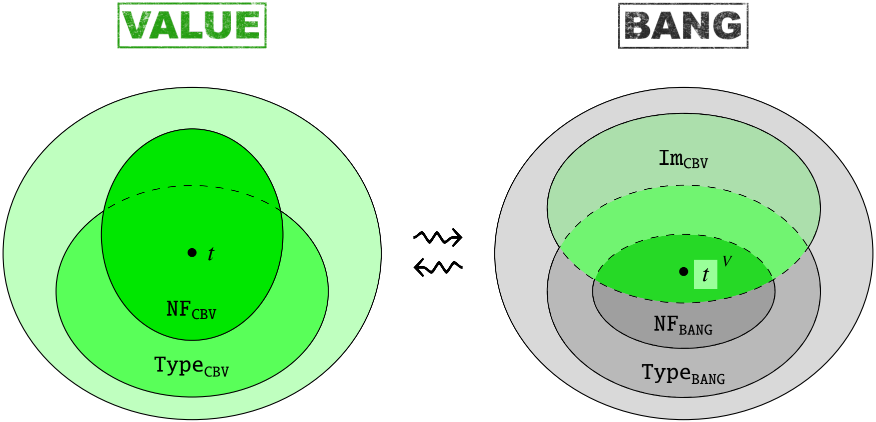

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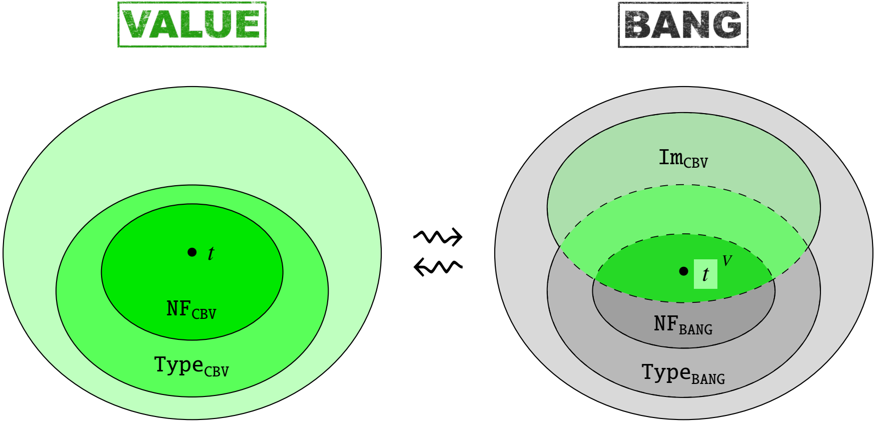

Usual Story with Intersection Types

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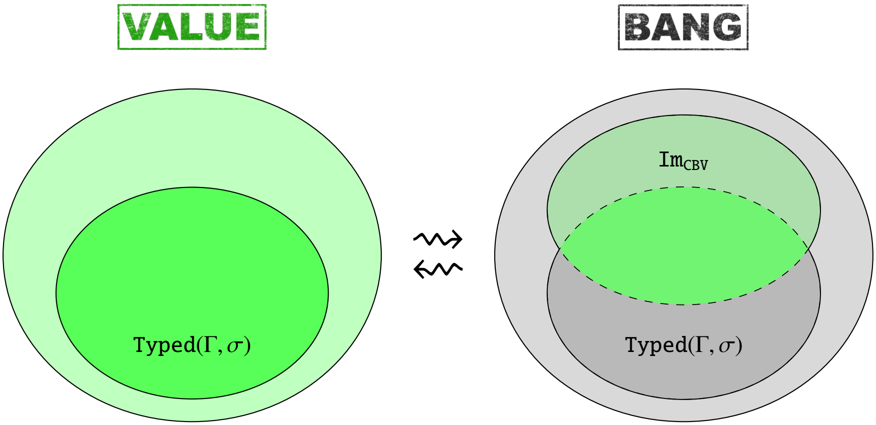

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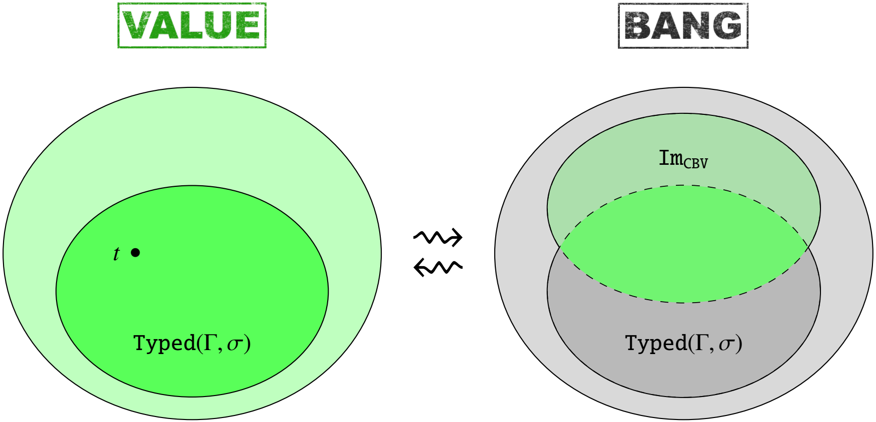

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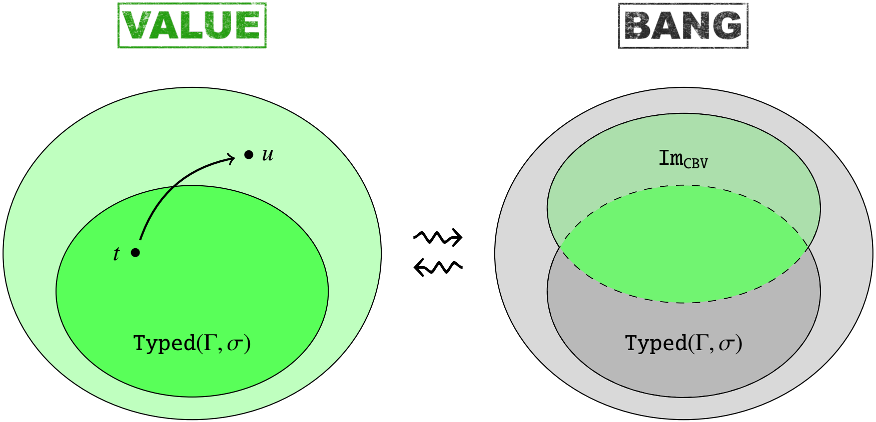

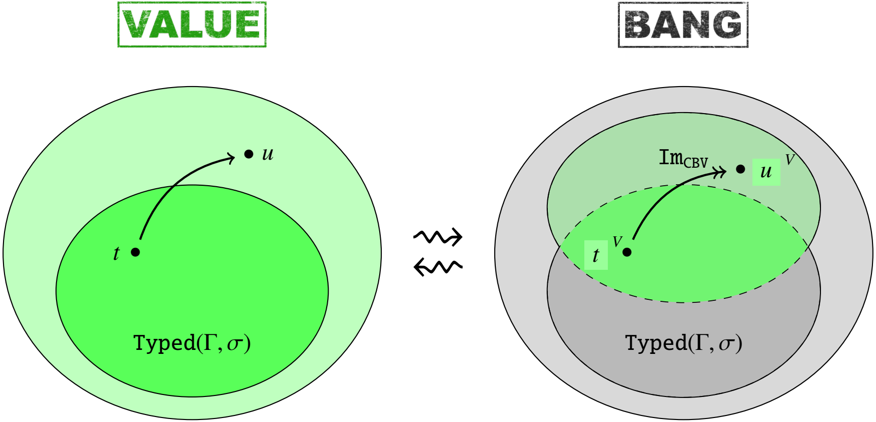
VALUE
BANG
u
t
Typed(Γ, σ)
$\mathtt{Im}_{\mathtt{CBV}}$
V
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Typed(Γ, σ)

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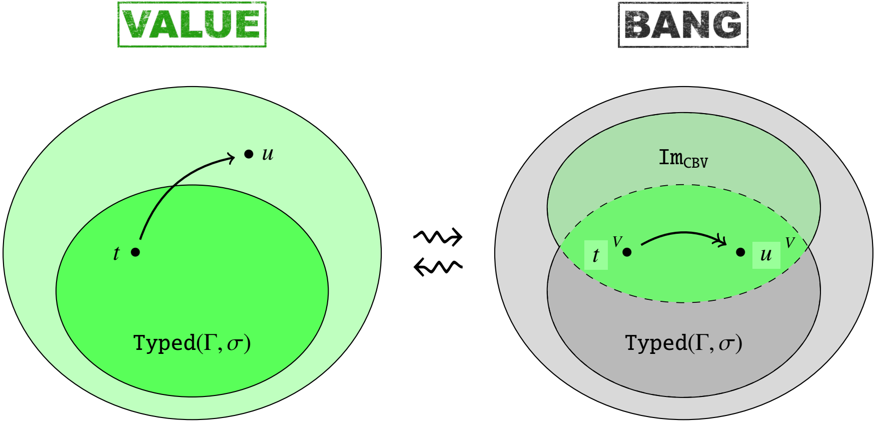

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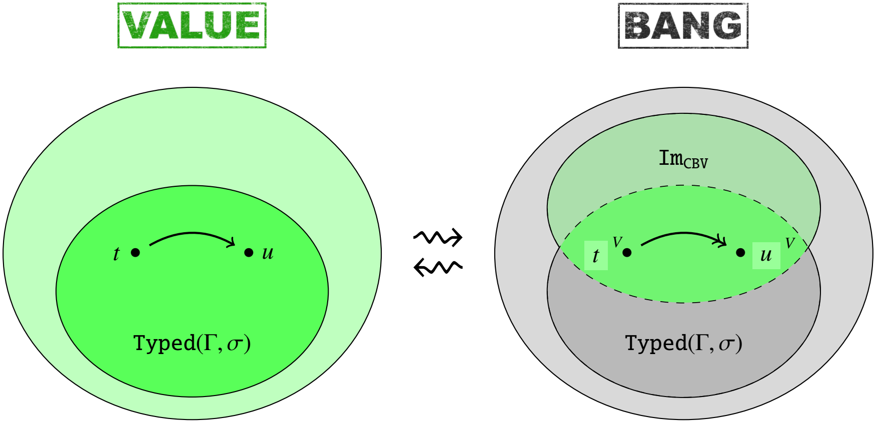

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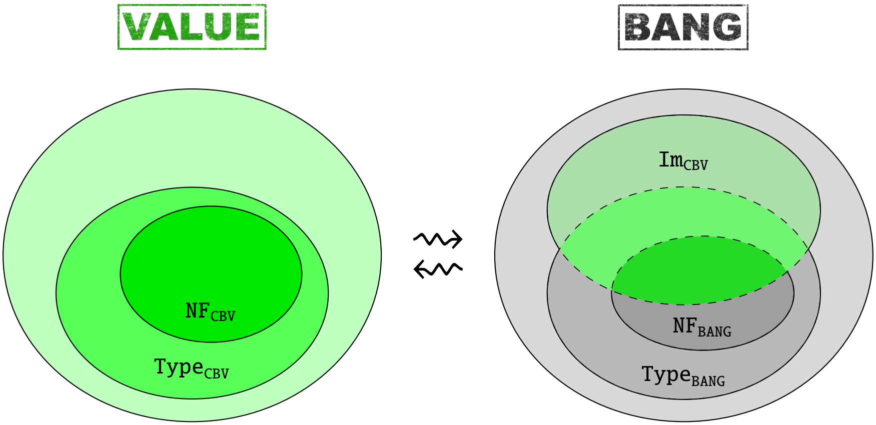

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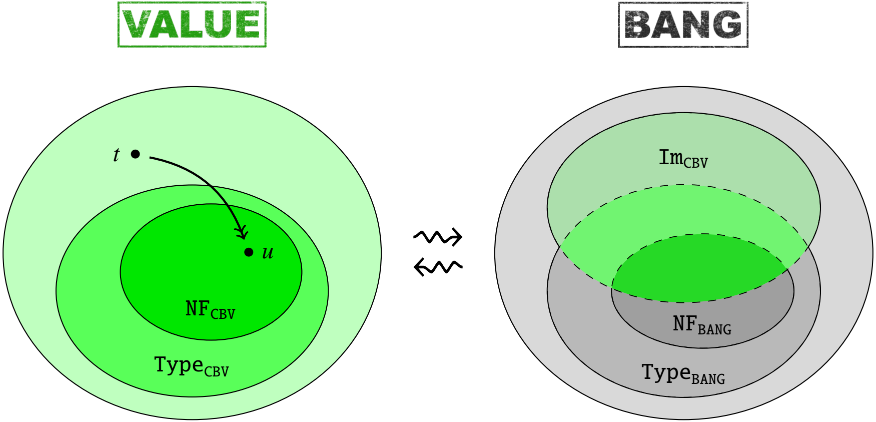

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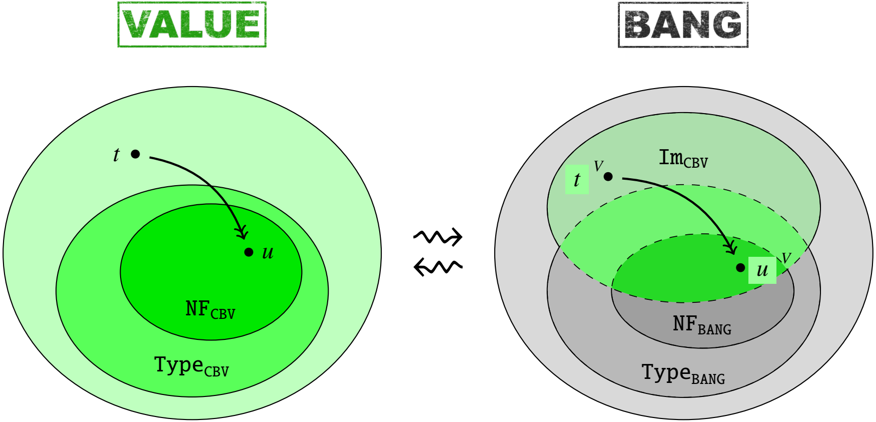

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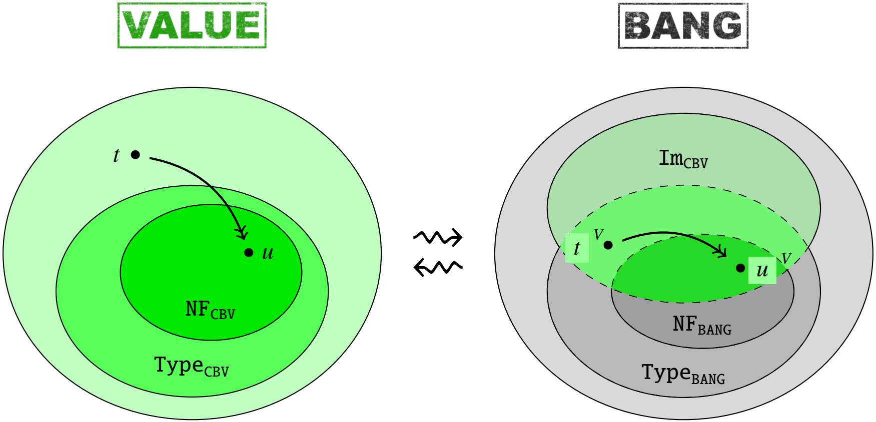

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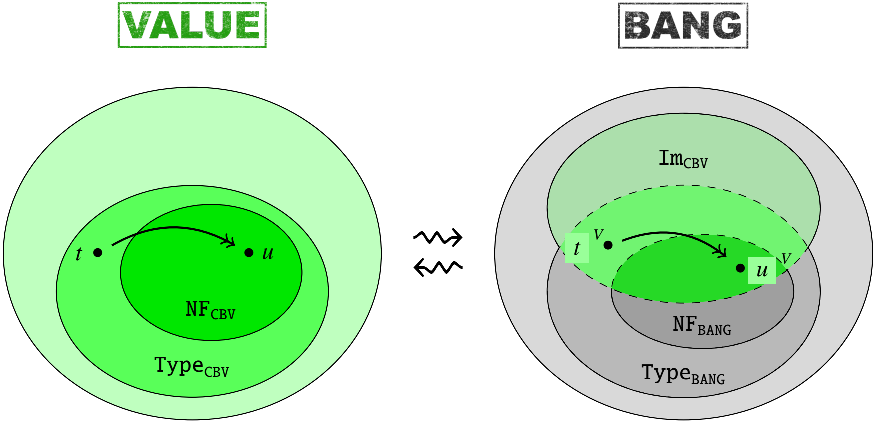

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- Preservation of typing
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Thank you !